

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	Use different objects to add equal groups	Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 3+3+3+3+3 = 15	Write addition sentences to describe objects and pictures. 2+2+2+2 = 10
Understanding ar- rays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.	Draw representations of arrays to show under- standing	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$

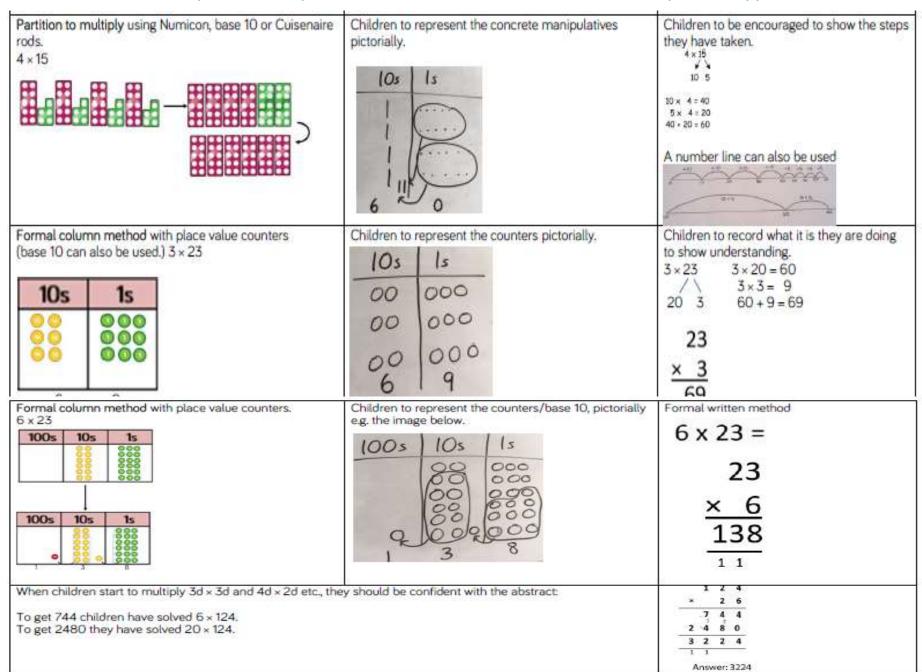
Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling	Double 4 is 8	Partition a number and then double each part before recombining it back together. 16 10 6 1x2 1x2 20 + 12 = 32
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. 5+5+5+5+5+5+5+5+5=40	Number lines, counting sticks and bar models should be used to show representation of counting in multiples. 3 3 3 3 3	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30



Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon. Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	Use representations of arrays to show different calculations and explore commutativity.	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse This should be taught alongside division, so pupils learn how they work alongside each other.		8	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8 ÷ 2 Show all 8 related fact family sentences.







Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.
			3 · 1 9
			× 8 25 · 52



Objective & Strategy	Concrete	Pictorial	Abstract
Strategy Division as sharing Use Gordon ITPs for modelling		Children use pictures or shapes to share quantities. 8 Strateu Detween 2 15 4 Sharing: 12 shared between 3 is 4	12 shared between 3 is
	I have 10 cubes, can you share them equally in 2 groups?		





Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 8 + 2 = 4 Children use bar modelling to show and support understanding.	12 ÷ 3 = 4
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use number lines for grouping 12 ÷ 3 = 4 Think of the parads a whole, split it into the number of groups you are dividing by and work out how many would be within each group. 20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?





Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding. 24 divided into groups of $6 = 4$ 96 ÷ 3 = 32	Continue to use bar modelling to aid solving division problems. 20 20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4





Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	Divide objects between groups and see how much is left over Example without 40 + 5 Ask "How many Example with re 38 + 6 For larger number jumps can be recommended."	5s in 40?" 5+5+5+5+5+5+5 = 8 f	a remainder of 2





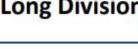
Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	3 2 3 Use place value counters to divide using the bus stop method alongside 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 2 1 8 3 4 8 7 2 Move onto divisions with a remainder. 8 6 r 2 3 5 4 3 2 Finally move into decimal places to divide the total accurately. 1 4 6 16 21 3 5 5 1 1 . 0

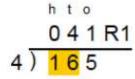




Long Division







4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

- 8 goes into 32 four times (3,200 + 8 = 400)
- 8 goes into 0 zero times (tens).
- 8 goes into 7 zero times, and leaves a remainder of 7.





Long Division



Step 1 continued...

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$



Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o	t o	t o
2) <mark>5</mark> 8	2)58 -4	2 9 2) 5 8 - 4 1
Two goes into 5 two times, or 5 tens + 2 = 2 whole tens but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o	t o	t o
2 9 2) 5 8	2 9 2 9 5 8	2)58
<u>-4</u>	<u>-4</u>	- <u>4</u> 18
10	-18	<u>- 18</u>
	0	0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.





Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
1 2) 2 7 8	1 2)278 -2 0	1 8 2)278 -21 07
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	Multiply & subtract.	Drop down the next digit.
h t o 13 2)278 -2 07 Divide 2 into 7. Place 3 into the quotient.	$\begin{array}{c} h \text{ to} \\ 13 \\ 2)278 \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \\ \end{array}$ Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 13 2)278 -2 07 -6 18 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
13 <mark>9</mark> 2)278 -2 07 -6 18	139 2)278 -2 07 -6 18 -18	2)278 -2 07 -6 18 -18
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.

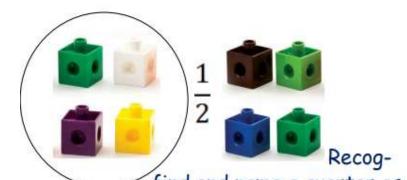




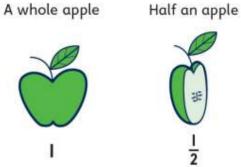
Year

Recognise, find and name a half as one of two equal parts of an object, shape or quantity.

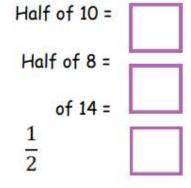




Pictorial



Abstract



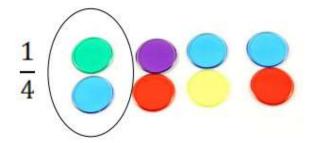
nise,

one

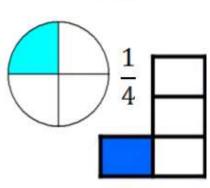
find and name a quarter as

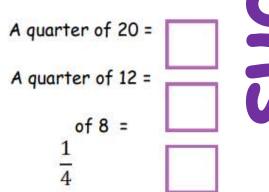
four equal parts of an object, shape or quantity.

Concrete



Pictorial

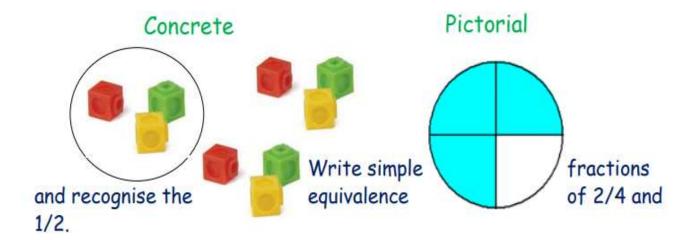




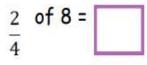
Year

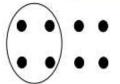
Recognise, find and name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity.

2



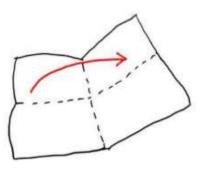




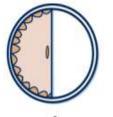


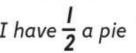
ractions

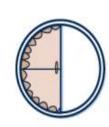
Concrete



Pictorial

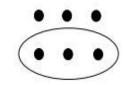






I have $\frac{1}{2}$ a pie You have $\frac{2}{4}$ of a pie

$$\frac{1}{2}$$
 of 6 =



Count up and down in tenths: recognise that tenths arise from dividing an object into ten equal parts and in dividing one-digit numbers or quantities by ten.

Year

3

Concrete 10 $\frac{5}{10}$ $\frac{6}{10}$ 10

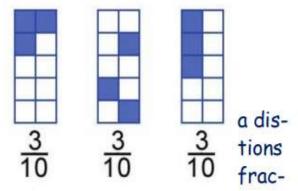
crete set of and non-unit tions as num-

Recognise, find and write fractions of objects: unit fracfractions and use bers.

10

7

Pictorial



Abstract

$$\frac{1}{10} \text{ of } 6 = 0.6$$
because
$$6 \div 10 = 0.6$$

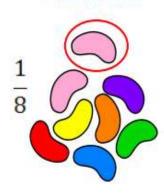
$$\frac{1}{10} \text{ of } 7 = 0.7$$

$$\frac{1}{10} \text{ because}$$

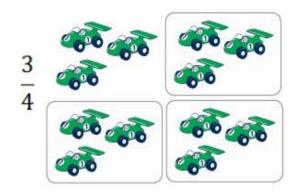
$$7 \div 10 = 0.7$$

tions

Concrete



Pictorial



$$\frac{1}{5}$$
 of 15 sweets = 3
= 2 cause 15 ÷ 5 = 3

$$\frac{2}{5}$$
 of 15 sweets = 6
becai $\frac{5}{5}$ 15 ÷ 5 = 3 and 3 x 2 = 6

Pictorial

Recognise and show, using diagrams, equivalent fractions with small denominators.

Year

3

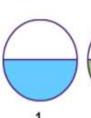
Concrete

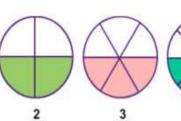


two halves

four quarters

Add sub-







4 8

Abstract

Sam says that two quarters is the same as one half.

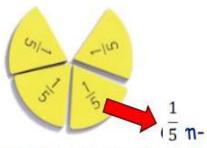
Is he correct?

How do you know?

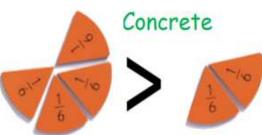
and

tract fractions with the same denominator.

Concrete



and order unit fractions the same denominators.



Pictorial



Pictorial



Abstract

$$\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$$

$$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$
 pare with

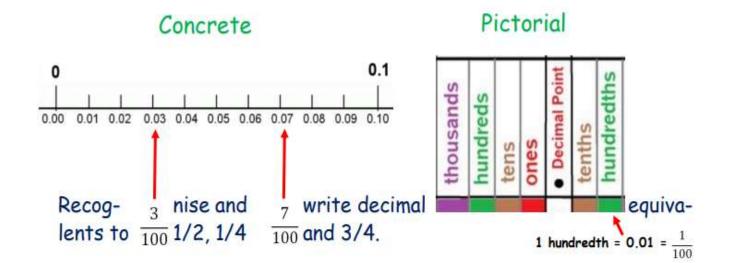


2	3	5	7	
8	8	8	8	

Count up and down in hundredths: recognise that hundredths arise when dividing an object by 100 and dividing tenths by 10.

Year

4

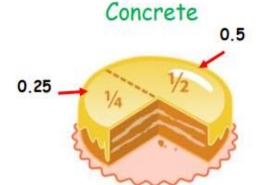


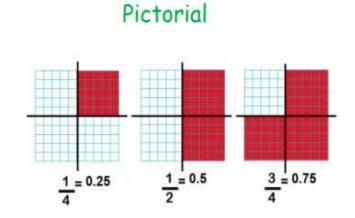
Abstract

$$\frac{1}{100}$$
 of 60 = 0.6
because 60 ÷ 100 = 0.6

of 70 = 0.7
so
$$\frac{1}{100}$$
 of 70 = 0.07

ractions





$$\frac{1}{2} = 0.5$$

$$\frac{1}{4} = 0.25$$

$$\frac{1}{4} = 0.75$$

$$\frac{3}{4}$$

Concrete

Pictorial

$$\frac{1}{10}$$
 = 0.1

Abstract

$$\frac{3}{10}$$
 = 0.3

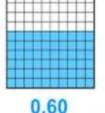
$$\frac{5}{10} = \frac{1}{2} = 0.5$$



Rec- $\frac{1}{10}$ of the chocolate bar = 0.1



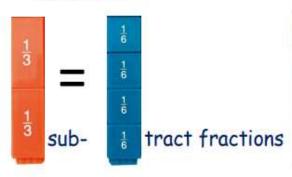
six tenths



sixty hundredths and show,

grams, families of common equivalents.

Concrete



ognise

Pictorial

				440	ille.				
1 2					3-				
1 1				1/3					
	1	Т		1 4		Ŧ	П	1	
- 1		Г	ŧ	1			ŧ		
+	T	+	Т	1 6	1 6		$\frac{1}{6}$	Т	÷
+	1 0		100	+	+			1	+
10	10	10	1	10	10	10	10	10	10
			-	6 4	1	+	-		15

with the same

Abstract

$$\frac{2}{3} = \frac{4}{6}$$

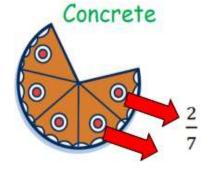
$$\frac{3}{5} = \frac{6}{10}$$

$$\frac{2}{12} = \frac{1}{6}$$

denomi-

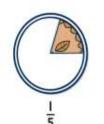
Abstract

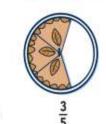
Sam eats 2 of a whole pizza. How much 7 oes he have left?



Add and

nator.





Pictorial



Lucy and Ben both eat of a cake. How much have they eat $\frac{3}{8}$ altogether?

Year

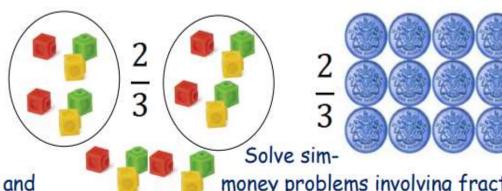
actions

Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number.

Year

4

Pictorial Concrete



ple meas-

money problems involving fractions and decimals to

Abstract

$$\frac{2}{3}$$
 of £18

£18 ÷
$$3 = £6$$

two

actions

Concrete

decimal places.



Pictorial

U		t	h	
Units	Decimal Point	Tenths	Hundredths	

$$50cm = \frac{1}{2} = 0.5m$$

$$25cm = \frac{1}{4} = 0.25m$$

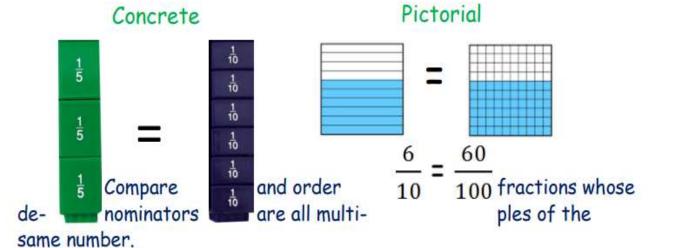
$$10cm = \frac{1}{10} = 0.1m$$

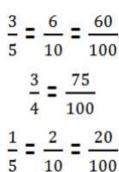
$$30cm = \frac{3}{10} = 0.3m$$

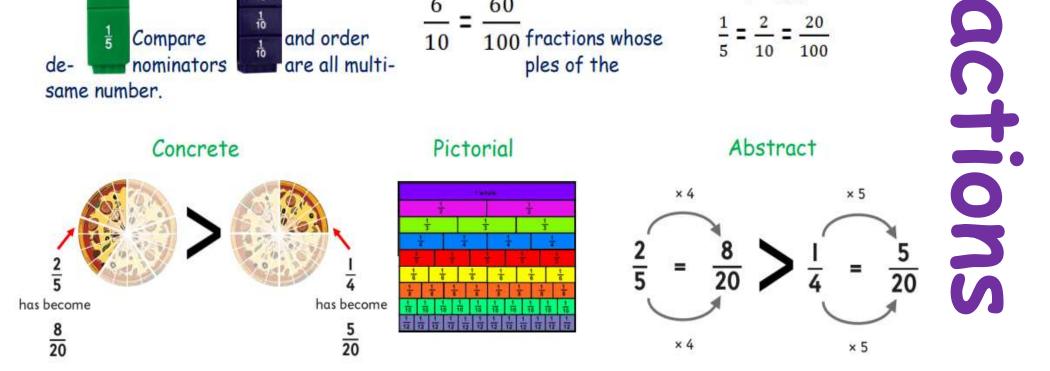
Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths.

Year

5



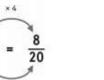




Recognise mixed numbers and improper fractions. Convert from one form to the other and write mathematical statements >1 as a mixed number.

Pictorial

Year





 $\frac{8}{20}$

$$\frac{1}{4}$$

Abstract

because 7 ÷ 2 = 3 with 1 half left over

because 2 x 3 = 6 with 1 third left to add

Abstract

Concrete

Concrete

Add

and

and denominators that are multiples of the same numbers.

Pictorial

subtract fractions with the same denominators

 $\frac{7}{5}$

Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

Year

Concrete

Pictorial

Abstract

5













Multiply a proper fraction by a whole number

Change to a mixed number:

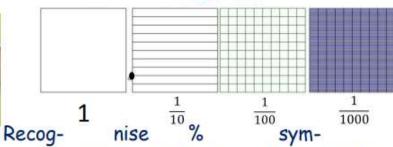
6 lots of $\frac{3}{4}$ tenths, hundredths and to

Recognise and use $4\frac{2}{4}$ altogether thousandths and relate $\frac{18}{4} = 4\frac{2}{4}$ them decimal equivalents.

Concrete



Pictorial



Abstract

67,153

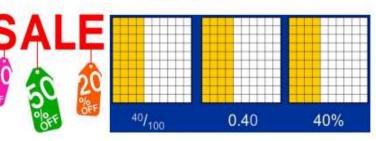
How many thousandths does this number have? How many more thousandths do you need to add to make 67.16?

bol and understand the meaning: write % as a fraction, decimal and percentage.

Concrete



Pictorial



$$\frac{4}{10}$$
 = 40% = 0.4

$$\frac{32}{100}$$
 = 32% = 0.32

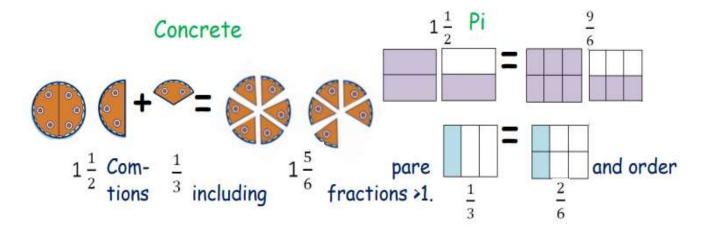
$$\frac{75}{100}$$
 = 75% = 0.75

$$\frac{2}{25} = \frac{8}{100} = 8\% = 0.08$$

Year

Add and subtract fractions with different denominators and mixed numbers using the concept of equivalent fractions.

6



Abstract

$$1\frac{1}{2} + \frac{1}{3} = 1\frac{5}{6}$$

because
$$1\frac{1}{2} = \frac{3}{2}$$

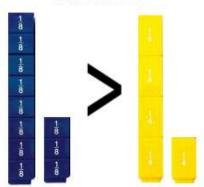
frac-

$$\frac{3}{2} = \frac{9}{6}$$
 and $\frac{1}{3} = \frac{2}{6}$

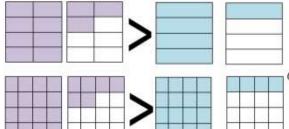
$$so \frac{9}{6} + \frac{2}{6} = \frac{11}{6} = 1\frac{5}{6}$$

actions





Pictorial



Abstract

Which is greater?

$$\frac{2}{8} < \frac{6}{16}$$

Ordering from smallest to largest by using equivalent fractions:

$$\frac{5}{12}$$
, $\frac{2}{3}$, $\frac{5}{6}$

$$\frac{5}{12}$$
, $\frac{8}{12}$, $\frac{10}{12}$

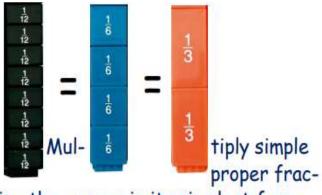
Use common factors to simplify fractions; use common multiples to express fractions in the same denomination.

Year

6

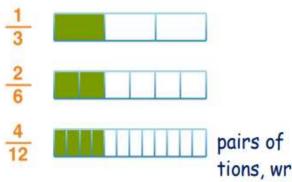
actions

Concrete



ing the answer in its simplest form.

Pictorial



tions, writ-

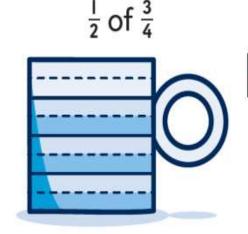
÷ 3 ÷ 6

Abstract

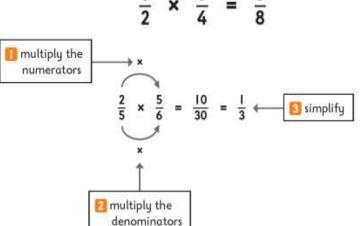
Abstract

$\frac{1}{2}$ of $\frac{3}{4}$

Concrete



Pictorial



Recall and use equivalences between simple fractions, decimals and percentages including in different contexts.

Year

6

Abstract

John scored $\frac{40}{80}$ in his spelling test and Hannah scored 40%. Who scored more?

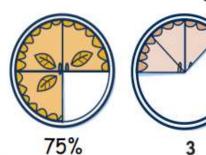
One paving slab is 0.3m long and another of a metre. Which is longer? = 0.25m0.3m is larger than 0.25m

Concrete



Pictorial

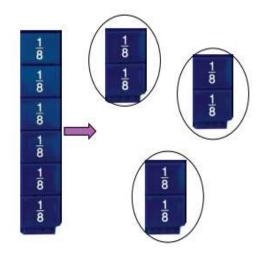
Which would you prefer 75% or $\frac{3}{8}$ of a pie?



Divide proper by whole numbers.

fractions

Concrete



Pictorial



$$\frac{1}{2} \div 3 = \frac{1}{6}$$

Abstract

$$\frac{1}{2} \div 3 = \frac{1}{6}$$

Keep it, change it, flip it!

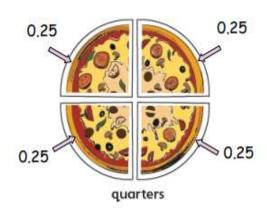
$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Year

6

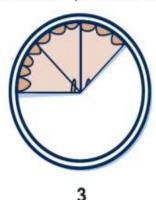
Associate fractions with division and calculate decimal fraction equivalents.

Concrete



Pictorial

3 slices of pie 'out of' 8



Abstract

3 8

3 'out of' 8 is the same as 3 'divided by' 8

$$3 \div 8 = 0.375$$

$$So \frac{3}{8} = 0.375$$

